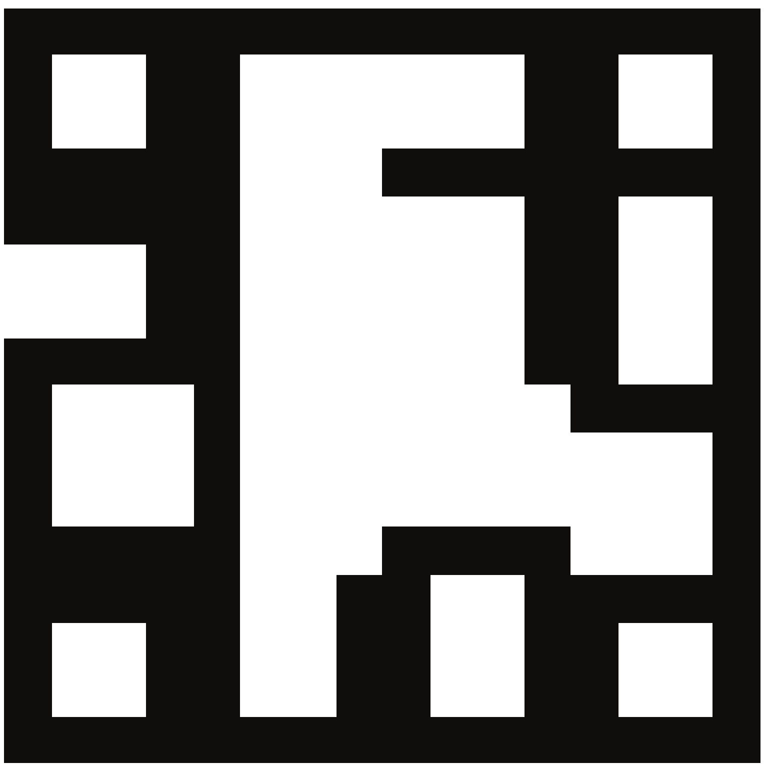
Wiley Finance Series

The Heston Model

and Its Extensions in Matlab and C#



Fabrice Douglas Rouah

FOREWORD BY STEVEN L. HESTON

WILEY

Foreword

I am pleased to introduce The Heston Model and its Extensions in Matlab and C# by Fabrice Rouah. Although I was already familiar with his previous book entitled Option Pricing Models and Volatility Using Excel/VBA, I was pleasantly surprised to discover he had written a book devoted exclusively to the model that I developed in 1993 and to the many enhancements that have been brought to the original model in the twenty years since its introduction. Obviously this focus makes the book more specialized than his previous work. Indeed, it contains detailed analyses and extensive computer implementations that will appeal to careful, interested readers. This book should interest a broad audience of practitioners and academics, including graduate students, quants on trading desks and in risk management, and researchers in option pricing and financial engineering.

There are existing computer programs for calculating option prices, such as those in Rouah's prior book or those available on Bloomberg systems. But this book offers more. In particular, it contains detailed theoretical analyses in addition to practical Matlab and C# code for implementing not only the original model, but also the many extensions that academics and practitioners have developed specifically for the model. The book analyzes numerical integration, the calculation of Greeks, American options, many simulation-based methods for pricing, finite difference numerical schemes, and recent developments such as the introduction of time-dependent parameters and the double version of the model. The breadth of methods covered in this book provides comprehensive support for implementation by practitioners and empirical researchers who need fast and reliable computations.

The methods covered in this book are not limited to the specific application of option pricing. The techniques apply to many option and financial engineering models. The book also illustrates how implementation of seemingly straightforward mathematical models can raise many questions. For example, one colleague noted that a common question on the Wilmott forums was how to calculate a complex logarithm while still guaranteeing that the option model produces real values. Obviously an imaginary option value will cause problems in practice! This book resolves many similar difficulties and will reward the dedicated reader with clear answers and practical solutions. I hope you enjoy reading it as much as I did.

Professor Steven L. Heston
Robert H. Smith School of Business
University of Maryland
January 3, 2013

Preface

In the twenty years since its introduction in 1993, the Heston model has become one of the most important models, if not the single most important model, in a then-revolutionary approach to pricing options known as stochastic volatility modeling. To understand why this model has become so important, we must revisit an event that shook financial markets around the world: the stock market crash of October 1987 and its subsequent impact on mathematical models to price options.
The exacerbation of smiles and skews in the implied volatility surface that resulted from the crash brought into question the ability of the Black-Scholes model to provide adequate prices in a new regime of volatility skews, and served to highlight the restrictive assumptions underlying the model. The most tenuous of these assumptions is that of continuously compounded stock returns being normally distributed with constant volatility. An abundance of empirical studies since the 1987 crash have shown that this assumption does not hold in equities markets. It is now a stylized fact in these markets that returns distributions are not normal. Returns exhibit skewness, and kurtosis—fat tails—that normality cannot account for. Volatility is not constant in time, but tends to be inversely related to price, with high stock prices usually showing lower volatility than low stock prices. A number of researchers have sought to eliminate this assumption in their models, by allowing volatility to be time-varying.
One popular approach for allowing time-varying volatility is to specify that volatility be driven by its own stochastic process. The models that use this approach, including the Heston (1993) model, are known as stochastic volatility models. The models of Hull and White (1987), Scott (1987), Wiggins (1987), Chensey and Scott (1989), and Stein and Stein (1991) are among the most significant stochastic volatility models that pre-date Steve Heston's model. The Heston model was not the first stochastic volatility model to be introduced to the problem of pricing options, but it has emerged as the most important and now serves as a benchmark against which many other stochastic volatility models are compared.
Allowing for non-normality can be done by introducing skewness and kurtosis in the option price directly, as done, for example, by Jarrow and Rudd (1982), Corrado and Su (1997), and Backus, Foresi, and Wu (2004). In these models, skewness and kurtosis are specified in Edgeworth expansions or Gram-Charlier expansions. In stochastic volatility models, skewness can be induced by allowing correlation between the processes driving the stock price and the process driving its volatility. Alternatively, skewness can arise by introducing jumps into the stochastic process driving the underlying asset price.
The parameters of the Heston model are able to induce skewness and kurtosis, and produce a smile or skew in implied volatilities extracted from option prices generated by the model. The model easily allows for the inverse relationship between price level and volatility in a manner that is intuitive and easy to understand. Moreover, the call price in the Heston model is available in closed form, up to an integral that must be evaluated numerically. For these reasons, the Heston model has become the most popular stochastic volatility model for pricing equity options.

Another reason the Heston model is so important is that it is the first to exploit characteristic functions in option pricing, by recognizing that the terminal price density need not be known—, only its characteristic function. This crucial line of reasoning was the genesis for a new approach for pricing options, known as pricing by characteristic functions. See Zhu (2010) for a discussion.

In this book, we present a treatment of the classical Heston model, but also of the many extensions that researchers from the academic and practitioner communities have contributed to this model since its inception. In Chapter 1 we derive the characteristic function and call price of Heston's (1993) original derivation. Chapter 2 deals with some of the issues around the model such as integrand discontinuities, and also shows how to model implied and local volatility in the model. Chapter 3 presents several Fourier transform methods for the model, and Chapter 4 deals exclusively with Alan Lewis' (2000, 2001) approach to stochastic volatility modeling, as it applies to the Heston model. Chapter 5 presents a variety of numerical integration schemes and explains how integration can be speeded up. Chapter 6 deals with parameter estimation and Chapter 7 presents classical simulation schemes applied to the model, and several simulation schemes designed specifically for the model. Chapter 8 deals with pricing American options in the Heston model. Chapter 9 presents models in which the parameters of the original Heston model are allowed to be piecewise constant. Chapter 10 presents methods for obtaining the call price that rely on solving the Heston partial differential equation with finite differences. Chapter 11 presents the Greeks in the Heston model. Finally, Chapter 12 presents the double Heston model, which introduces an additional stochastic process for variance and thus allows the model to provide a better fit to the volatility surface.

All of the models presented in this book have been coded in Matlab and C#.

Acknowledgments

I would like to thank Steve Heston not only for having bestowed his model to the financial engineering community, but also for contributing the Foreword to this book and to Leif B.G. Andersen, Marco Avellaneda, Peter Christoffersen, Jim Gatheral, Espen Gaarder Haug, Andrew Lesniewski, and Alan Lewis for their generous endorsement. And to my team at Wiley—Bill Falloon, Meg Freeborn, Steven Kyritz, and Tiffany Charbonier—thank you. I am also grateful to Gilles Gheerbrant for his strikingly beautiful cover design.

Special thanks also to a group who offered moral support, advice, and technical reviews of the material in this book: Amir Atiya, Carl Chiarella, Elton Daal, Redouane El-Kamhi, Judith Farer, Jacqueline Gheerbrant, Emmanuel Gobet, Greg N. Gregoriou, Antoine Jacquier, Dominique Legros, Pierre Leignadier, Alexey Medvedev, Sanjay K. Nawalkha, Razvan Pascalau, Jean Rouah, Olivier Scaillet, Martin Schmelzle, and Giovanna Sestito. Lastly, a special mention to Kevin Samborn at Sapient Global Markets for his help and support.

Endorsements

In this thorough account of the Heston Model -- the fundamental model for volatility used in the market -- Fabrice Rouah gives a 360-degree view of implementation and estimation issues. The book covers the Heston PDE, quadrature methods, the Fourier transform approach, simulation methods and model estimation. It includes Matlab and C# code covering the material. Without a doubt, Fabrice provides a very valuable contribution to quantitative analysts interested in pricing equity, FX and interest rate options with state-of-the art techniques.
Marco Avellaneda
Professor of Mathematics, Courant Institute, New York University
2010 Risk Magazine Quant of the Year

The Heston model is one of the great success stories of academic finance. It provides important insights in its own right and it has served as the building block for extensions in a plethora of articles published since Heston's original work. Dr. Rouah's impressive book provides users with all the tools required to implement the Heston model, including integral computation, parameter estimation, latent factor simulation and American option pricing. These tools are typically not detailed in academic articles and Fabrice's book therefore wonderfully bridges the gap between academia and practice.
Peter Christoffersen
Professor of Finance, Rotman School of Management, University of Toronto

This is the most extensive work on the Heston model I have seen; derivations, implementations, and discussions. For anyone interested in the Heston model and its variations this is an important book to have!
Espen Gaarder Haug
Professor of Finance, Norwegian University of Life Sciences
Author of "Derivatives Models on Models"

In this encyclopedic work, the author takes evident delight in exploring every aspect of the Heston model from derivation to implementation. From Lord and Kahl's optimal choice of the Carr Madan damping factor in the Heston integration to the method of Medvedev and Scaillet for approximating American option prices, there is a wealth of interesting and useful material here that is otherwise not widely known or hard to access. Together with its included Matlab and C# code, this book will prove invaluable to anyone interested in option pricing, whether academic or practitioner. I highly recommend it.
Jim Gatheral
Presidential Professor, Baruch College CUNY
Author of "The Volatility Surface: A Practitioner's Guide"

Fabrice Rouah offers a unique and much needed synthesis of the literature regarding Heston's model of stochastic volatility. The author has accomplished the formidable task of presenting a large body of published academic and industrial research in a coherent, thorough, and very reader-friendly manner. Each concept discussed in the book is illustrated with carefully designed computer code written in Matlab and C#. Rouah's book is an indispensable tool for all financial professionals interested in obtaining a deep understanding of modeling and risk management of financial derivatives.
Andrew Lesniewski
Head of Financial Engineering, DTCC

Beyond Black-Scholes, the Heston model is arguably the most important model in quantitative finance and certainly deserves its own book. Rouah provides here a comprehensive treatment -- clearly discussing all the major issues, later extensions, and subtle traps -- with Matlab and C# code as well. Students, practitioners, and researchers will want to acquire this one stop source.
Alan L. Lewis, PhD
Author of "Option Valuation Under Stochastic Volatility: With Mathematica Code"

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