One question that is often asked of the Black-Scholes formula is "I know that \( \Phi(d_2) \) is the probability of the call ending up in-the-money, but what does \( \Phi(d_1) \) represent, in terms of a probability?". The answer is that \( \Phi(d_1) \) and \( \Phi(d_2) \) are both probabilities of the call ending up in-the-money, but under different measures.

Recall that according to the Black-Scholes model, the stock price is driven by the following SDE

\[
dS_t = rS_t dt + \sigma dW^P_t
\]

where \( P \) is the risk neutral measure under which \( W^P_t \) is Brownian motion. The Black-Scholes call price with maturity \( T-t \) and strike \( K \) is

\[
C = e^{-r(T-t)} E^P \left[ (S_T - K)^+ \right] = e^{-r(T-t)} E^P \left[ (S_T - K) \mathbf{1}_{S_T > K} \right]
\]

(1)

where \( \mathbf{1}_{S_T > K} \) is the indicator function, and where the expectation is taken at time \( t \). This can be written as

\[
C = e^{-r(T-t)} E^P \left[ S_T \mathbf{1}_{S_T > K} \right] - K e^{-r(T-t)} E^P \left[ \mathbf{1}_{S_T > K} \right]
\]

(2)

Equation (2) is a general expression for the call price when interest rates are constant. In Equation (2), \( E^P \left[ \mathbf{1}_{S_T > K} \right] = \mathbb{P} (S_T > K) \) and in the special case of Black-Scholes– namely, when \( S_T \) follows a lognormal distribution—it is equal to \( \Phi(d_2) \) so it’s easy to see that \( \Phi(d_2) \) is the probability of the call ending in-the-money. The second expectation is, writing \( B_t = e^{rt} \)

\[
e^{-r(T-t)} E^P \left[ S_T \mathbf{1}_{S_T > K} \right] = E^P \left[ \frac{B_t}{B_T} S_T \mathbf{1}_{S_T > K} \right] = S_t E^P \left[ \frac{B_t}{B_T} \frac{S_T}{S_t} \mathbf{1}_{S_T > K} \right].
\]

Now define the Radon-Nikodym derivative as the ratio of numeraires \( Z_T = \frac{dP}{dQ} = \frac{S_t}{B_t/B_T} \). Then the expectation can be changed from measure \( P \) to a new measure \( Q \) by using the Radon-Nikodym derivative as follows.

\[
E^P \left[ \frac{B_t/B_T}{S_t/S_T} \mathbf{1}_{S_T > K} \right] = E^Q \left[ \frac{B_t/B_T}{S_t/S_T} \mathbf{1}_{S_T > K} Z_T \right] = E^Q \left[ \mathbf{1}_{S_T > K} \right] = Q (S_T > K).
\]

This is the probability of exercise, but under measure \( Q \). Hence the call price in Equation (2) can be written

\[
C = S_t Q (S_T > K) - K e^{-r(T-t)} \mathbb{P} (S_T > K).
\]

(3)
where

\[ P(S_T > K) = \text{probability of exercise under the original measure } P \]
\[ Q(S_T > K) = \text{probability of exercise under the new measure } Q \]

In the Black-Scholes model, the call price is

\[ C = S_t \Phi (d_1) - K e^{-(r-t)T} \Phi (d_2). \]  

Comparing Equations (3) and (4), we see that in the Black-Scholes model, \( \Phi (d_2) = P(S_T > K) \) and \( \Phi (d_1) = Q(S_T > K) \).